Microscopic description of low-lying properties in $168 \leqslant A \leqslant 170$ Yb nuclei by the pseudo-SU(3) shell model

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Abstract

The rare-earth nuclei have well-known collective properties. The theoretical description of these nuclei represents a challenge to nuclear models, due to the enormous dimensions of the valence space, making the problem unmanageable. This leads us to use symmetry-based models, where it is possible to calculate in a free-truncation environment. In this work we present results for the energy spectrum and the electromagnetic properties in even-even Yb isotopes using the pseudo-SU(3) shell model. The model considers a Nilsson Hamiltonian that additionally includes the quadrupole-quadrupole and pairing interactions, systematically parameterized. The results show that the model considered is a powerful theoretical tool, allowing us to describe the normal parity sector of deformed rare earth nuclei.

Key words: *atomic models; rare earth nuclei; SU-3 groups; electrical properties; magnetic properties; ytterbium; energy spectra.*

Descripción microscópica de las propiedades de baja energía de los núcleos $168 \leqslant A \leqslant 170$Yb usando el modelo pseudo-SU(3)

Resumen

Las propiedades colectivas de los núcleos de las tierras raras están muy bien determinadas. La descripción teórica de estos núcleos representa un desafío para cualquier modelo nuclear debido a las dimensiones excesivas del espacio de valencia, lo que vuelve el problema inmanejable. Esto nos lleva a emplear modelos basados en simetrías, donde es posible realizar cálculos sin truncamiento del espacio. En este trabajo se presentan resultados para el espectro energético y las propiedades electromagnéticas en isótopos para-par de iterbio, utilizando el modelo de capas pseudo-SU(3). El modelo considera el hamiltoniano de Nilsson, al que adicionalmente se le han incluido las interacciones cuadrupolo-cuadrupolo y de apareamiento, parametrizadas de forma sistemática. El resultado muestra que el modelo utilizado es una herramienta poderosa que permite describir el sector de paridad normal del espectro de núcleos deformados de las tierras raras.

Palabras clave: *modelos atómicos; núcleos de tierras raras; grupos SU-3; propiedades eléctricas; propiedades magnéticas; iterbio; espectros de energía.*

Introduction

Singular aspects of the nuclear structure around the midshell region can be studied in the rare-earth neutron-rich nuclei. Between these, we can mention new modes of excitation and collectivity, the role of the single-particle levels and even possible modifications of the shell structure. However, the rare-earth region of the nuclear chart represents a great challenge, because the combination of stable projectiles and stable targets has unfavorable kinematic matching conditions leading to heavy neutron-rich nuclei, resulting in small cross-sections [1]. From the theoretical point of view, the large valence spaces associated with heavy nuclei have implied a slow progress, limiting the number of models capable of dealing with neutron rich rare-earth nuclei.

The pseudo-SU(3) model [2] has been used to describe normal parity bands in heavy deformed nuclei. The scheme takes full advantage of the existence of pseudospin symmetry, which refers to the fact that single-particle orbitals with $j=l-1/2$ and $j=(l-2)+1/2$ in the η shell lie close in energy and can therefore be labeled as pseudospin doublets with quantum numbers $\tilde{j}=j$, $\tilde{\eta}=\eta-1$ and $\tilde{l}=l+1$. The origin of this symmetry has been traced back to the relativistic Dirac equation [3]. In the simplest version of the pseudo-SU(3) model, the intruder level with opposite parity in each major shell is removed from active consideration and pseudoorbital and pseudospin angular momenta are assigned to the remaining single-particle states.

The first applications used pseudo-SU(3) as a dynamical symmetry, with a single irreducible representation irrep of SU(3) describing the yrast band up to the backbending regime [4]. On the computational side, the development of a computer code to calculate reduced matrix elements of physical operators between different SU(3) irreps [5] represented a breakthrough in the development of the pseudo- SU(3) model. With this code in place it is possible to include symmetry breaking terms in the interaction, such as spin-orbit and pairing terms which are known to represent important two-body correlations in low-energy configurations.

In the present paper we examine the ability of the pseudo-SU(3) model to describe interband B(E2) transition strengths in the even-even A=168 and 170 Yb nuclei.

Materials and methods

The selection of the many-body basis is the starting point for any shell-model application. Many-particle states of n active nucleons (for protons or neutrons) in a given (N) normal parity shell η are classified by the group chain described in Ref. [6]. The occupation numbers for protons are constant along the chain. In shell-model applications, the ytterbium isotopes are considered to have 20 protons out of the Z = 50 inert core, 12 of these in normal and 8 in abnormal h(11/2) parity levels. The occupation numbers for neutrons in Ytterbium 168 are 10 in normal and 6 in abnormal parity levels. In Ytterbium 170 they are 12 neutrons in normal and 6 in abnormal parity levels. As it has been the case for almost all studies with the model to date, nucleons in abnormal parity orbital are considered to renormalize the dynamics that is described using only nucleons in normal parity states. This limitation is reflected, for example, by the use of very high effective charges to describe quadrupole electromagnetic transitions.

The Hamiltonian contains spherical Nilsson single-particle terms for the protons and neutrons, the quadrupole-quadrupole and pairing interactions parametrized systematically [7], as well as three rotor-like terms that are diagonal in the SU(3) basis. A detailed analysis of each term of this Hamiltonian and its systematic parametrization can be found in Ref. [8]. The basic components of the Hamiltonian are the single-particle levels, pairing correlations and the quadrupole-quadrupole interaction, essential in the description of deformed nuclei. They have been widely studied in nuclear physics literature, allowing to fix their respective strengths by systematics (A dependent), consequently they are not considered as free parameters of the model. The SU(3) mixing is due to the symmetry-breaking Nilsson single-particle and pairing terms. An recent application of the present model to describe low-lying properties in Dy neutron-rich nuclei can be found in the Ref. [9]. In Table 1 are shown the parameters of the Hamiltonian used.

Table 1. Parameters (in keV) used in the Hamiltonian (Ref. [9], eq. (2))

Parameter	168Yb	170Yb
x	6.842	6.709
G_protons	125.0	123.5
G_neutrons	101.2	100.0
a	90.0	220.0
b	2.5	1.0
c	0.018	0.010

Results

Using the basis and Hamiltonian presented in the previous section, we present the results for ground-state, γ and β bands in the 168–170 Yb isotopes. In addition, we present inter-band B(E2) transition strengths. Table 2 shows the experimental [10] and theoretical energies for ground-state, γ and β bands in the 168-170 Yb nuclei.

Table 2. Experimental and theoretical energies (in keV) of ground-state, γ and β bands in 168–170 Yb nuclei

	168Yb		170Yb	
	Exp.	Theo.	Exp.	Theo.
0_g.s.b.	0	0	0	0
2_g.s.b.	87	85	84	80
4_g.s.b.	286	284	277	268
6_g.s.b.	585	594	573	564
2_γ	984	986	1145	1130
3_γ	1067	1101	–	1234
4_γ	1171	1250	1324	1378
5_γ	1302	1432	–	1540
6_γ	1445	1640	1601	1766
0_β	1154	1154	1069	1025
2_β	1233	1237	1138	1115
4_β	1391	1444	1292	1322
6_β	–	1781	1521	1648

The ground state band is very well described by the model in both nuclei, as can be seen in Table 2. For γ and β-bands we see that the model overestimates the energies.

In Table 3 we present inter-band B(E2) transition strengths. The total pseudo-spin content of the nuclear wave function is built through the coupling of the S_proton and S_neutron components. As it has been the case in previous works with the model, the ground state bands in the isotopes 168-170Yb are composed predominantly by S=0, with very small mixing of the S=1. The gamma and beta-bands have larger components of S=1. The pseudo-spin contents for each band are practically constant along all states of the band. These results show the importance of the S=1 contribution in the description of excited γ and β-bands.

Table 3. B(E2;J_i → J_f) inter-band transitions in 168-170Yb nuclei (given in e^2b^2 x 10^{-2})

	168Yb		170Yb	
J_i J_f	Exp.	Theo.	Exp.	Theo.
0_gs→ 2_β	4.96 +/– 0.55	0.001	3.02 +/– 0.59	0.041
0_gs→ 2_γ	13.77 +/– 1.93	0.009	7.55 +/– 1.68	0.001
0_ β→ 2_gs		*		0.159
0_β → 2_γ		0.006		151.4
2_gs→ 3_γ		0.002		0.009
2_gs→ 4_γ		*		0.052
2_gs→ 4_β		0.006		0.054
2_γ → 4_gs	0.99	0.002	0.27 +/– 0.06	0.109
2_γ→ 4_β		*		3.653
2_β → 3_γ		0.366		73.863
2_β → 4_gs		*		0.157
2_β → 4_γ		0.019		14.625

The first column gives the initial (J_i) and final (J_f) values of the angular momentum. From second to the fifth column the experimental (Exp.) and pseudo-SU(3) model calculations (Theo.) are shown. Effective charges are e_proton=2.3 and e_neutron=1.3

Discussion

An extended version of the pseudo-SU (3) model which includes pseudo-spin 0 and 1 states has been implemented to describe the B(E2) of positive-parity low-energy states in the rare-earth nuclei 168 - 170 Yb. By using a systematically parametrized Hamiltonian and the best fit of three parameters, the model has allowed the study of the spectra and B(E2). Most inter-band B(E2) transitions found in the present calculation are small compared with intra-band, showing that the wave function of the states are almost orthogonal. However, a few strengths are one order of magnitude bigger, indicating a greater mixing between ground- state, γ and β bands.

Conclusions

In the present contribution, the work has been focussed on 168-170 Yb isotopes. A logical continuation is to apply the model to heavier ytterbium nuclei, where the transition towards triaxiality could be more evident. An interesting continuation of the present work is the application of the model to the study of the nature of the 0+ excited states and its relation with the β excitations. Furthermore, the scheme employed in the present approach can also be used to describe electromagnetic properties in odd mass nuclei.

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