Alpha decay half-lives of superheavy nuclei in the WKB approximation

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Abstract
Alpha decay half-lives of superheavy nuclei are obtained in the context of barrier penetration theory built with the use of Coulomb and proximity potentials, taking into account the quadrupole deformations of nuclei. It is estimated from a classical viewpoint, a possible maximum value of the angular momentum of alpha particles emitted from odd and odd-odd nuclei. Masses and deformations of nuclei are obtained from the macro-microscopic method, with the use of the two-center shell model. Alpha-decay half-lives are compared with recent experimental results.

Períodos de semidesintegración alfa de núcleos superpesados en el marco de la aproximación WKB

Resumen
Se obtienen períodos de semidesintegración alfa en el marco de la teoría de penetración de barrera, esta última construida con el uso de los potenciales de proximidad y de Coulomb, teniendo en cuenta la deformación cuadrupolar de los núcleos. Se estima, desde el punto de vista clásico, el máximo valor posible del momento angular de las partículas alfa emitidas por núcleos impares e impar-impar. Las masas y las deformaciones de los núcleos se obtienen según el método macromicroscópico, con el uso del modelo de capas de dos centros. Los períodos de semidesintegración alfa se comparan con resultados experimentales.

Key Words: *alpha decay, transactinide elements, half-life, WKB approximation, potentials, ODD-ODD nuclei.*

Introduction

One of the main problems of modern nuclear physics is the extension of the periodic system into the islands of stability of superheavy elements (SHE). For the synthesis of these nuclei fusion-evaporation reactions are used and two approaches have been successfully employed: cold and hot fusion. The former have been used to produce new elements and isotopes up to Z = 113 [1,2]; the latter have been used to produce more neutron rich isotopes of elements up to Z = 118 [3]. The identification of SHEs in cold fusion reactions is based on the identification of the decay products via alpha correlations with known alpha emitters at the end of the decay sequences, but in hot fusion reactions the nuclei at the end of the decay sequences are neutron rich isotopes that have not been obtained yet in other kind of experiments; thus, in this type of reactions, alpha-decay half-lives systematics based on theoretical calculations provide a useful tool for an ulterior identification of the reaction products. Most of alpha decay half-lives calculations are performed with the aid of semi-empirical relationships [4-8]; alternatively, calculations in the framework of quantum mechanical tunneling have been done using the density-dependent M3Y interaction model [9,10], the proximity potential model [11] or using the relativistic mean-field model to calculate the interaction potential [12-14]. However, in most works the influence of deformed shapes of nuclei in the results of the half-lives calculations has been neglected. In this work is presented a method for obtaining alpha decay half-lives in the framework of WKB approximation using the proximity potential model, which takes into account quadrupole deformations of nuclei. Besides the fact that alpha-decay half-lives calculations can be used to identify new nuclei in experiments, they can be used as a way to test other theoretical results by comparison with experiment. In this work, new theoretical values of masses and deformations, calculated from the macro-microscopic method using the two center shell model, are used in order to obtain the alpha-decay half-lives, and the comparison of this half-lives values with experimental ones, can be useful to test the veracity of the calculation of masses and deformations.

Methods

Half-live Calculation

In the quantum tunneling theory of alpha decay, the decay constant λ can be expressed as the product of the alpha particle pre-formation probability P_0, by the number of assaults on the barrier per second n, by the barrier penetration probability *P*.

$$\lambda = P_0 \nu P \tag{1}$$

The half-live $T_{1/2}$, the main result of this paper, is related to the decay constant as

$$T_{1/2} = \ln\frac{2}{\lambda} \tag{2}$$

The barrier penetration probability is calculated using one-dimensional WKB approximation

$$P = \exp\left\{-\frac{2}{\hbar}\int_{z_1}^{z_2}\sqrt{2\mu\left(V(z)-Q_\alpha\right)}dz\right\} \tag{3}$$

where μ is the reduced mass. The potential energy *V* is the sum of the Coulomb V_C, nuclear V_N and centrifugal V_l energy.

$$V(z) = V_C(r) + V_N(z) + V_l(r) \tag{4}$$

In the above expressions, *z* and *r* are, respectively, the distances between the surfaces and between the centers of the alpha particle and the residual nucleus, both measured along an axis parallel to the vector which describes the relative motion; in the present work is considered that the alpha particle is emitted from the farthest point of the nuclear surface (see Fig. 1), because in this way the alpha particle face a lesser Coulomb barrier. The turning points z_1 and z_2 are determined from the equation

$$V(z_1) = V(z_2) = Q_\alpha \tag{5}$$

The barrier penetration probability *P*, is the most important factor in the half-live calculation, however, rather rough values of P_0 and ν can reduce significantly the accuracy of the results. For P_0 and ν, values in [8] have been taken; they have been obtained recently by a fit with a selected set of experimental data. For even-even, even-odd, odd-even and odd-odd nuclei we have

$$\begin{aligned} c_{ee} &= -20.198 \quad c_{eo} = -19.412 \\ c_{oe} &= -19.680 \quad c_{oo} = -18.903 \end{aligned} \tag{6}$$

where

$$c = \log_{10}(\ln(2)) - \log_{10}(P_0) - \log_{10}(\nu) \tag{7}$$

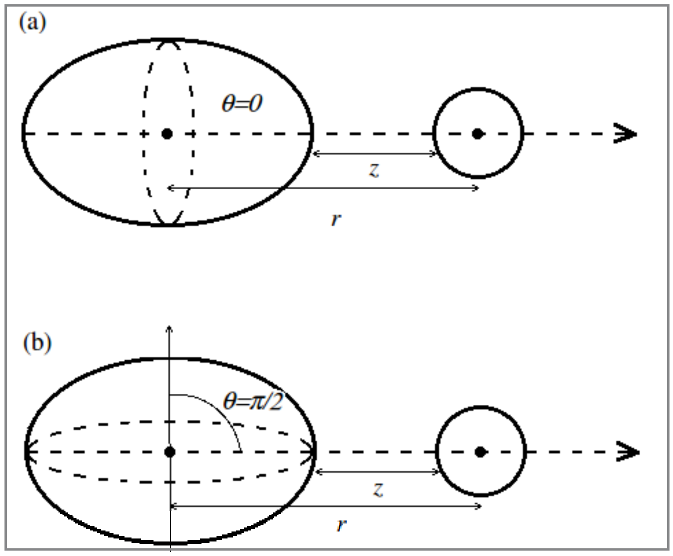

Figure 1: The alpha particle is emitted from the farthest point of the nuclear surface. (a) Case of prolate nucleus. (b) Case of oblate nucleus.

Proximity Potential

The nucleus is a leptodermous distribution, i. e., a distribution essentially homogeneous except for its surface. The strong attraction between two nuclei occurs when their surfaces approach to a distance comparable to the surface width *b*; the energy of this interaction can be described by the proximity potential [15].

$$V_N(z) = 4\pi\gamma b\overline{K}\Phi(z/b) \tag{8}$$

Here g is the nuclear surface tension coefficient, $\overline{K}$ is the reciprocal of the square root of the Gaussian curvature of the function that determines the distance between two points of the surfaces, evaluated at the point of closest approach, and Φ is an adimensional function called universal proximity potential.

The Lysekil formula is used for the surface tension coefficient [15].

$$\gamma = \gamma_0 \left(1 - 1.7826 I^2\right) \tag{9}$$

Here $\gamma_0 \approx 0.95$ MeV/fm^{-2} and $I = (N - Z)/A$, where N, Z and A refer to the set of both nuclei. Calculation of the Gaussian curvature of a function which depends on the shape of the surfaces of two deformed nuclei can be difficult; $\overline{K}$ in (8) can be replaced by a simple expression that depends on the principal curvatures k_i^x, k_i^y of the surfaces of both nuclei.

$$\overline{K}' = \left[\left(k_1^x + k_2^x\right)\left(k_1^y + k_2^y\right)\right]^{1/2} \tag{10}$$

For the expansion of the nuclear surface in spherical harmonics Y_l^m, generally are only taken into account quadrupole deformations; therefore, for a nucleus with axial symmetry, the radius R_N can be expressed as follows, depending on the parameter β_2:

$$R_N(\theta) = C\left(1 + \beta_2 Y_2^0(\theta)\right) \tag{11}$$

Here C is the radius of a spherical nucleus with the same volume as the deformed nucleus. To define the radius of a leptodermous distribution there are several parameters, the best known is the sharp radius, usually taken as $R = R_0 A^{1/3}$. However, when the proximity potential is used, it is preferable to take the radius of the nucleus as the central radius [16], which is determined mostly by the characteristics of the surface of the nucleus and not by the value of the density distribution function inside the nucleus. The central radius is related with the sharp radius by the expression:

$$C = R - \frac{b^2}{R} \tag{12}$$

The next formula can be used for the sharp radius

$$R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3} \tag{13}$$

as it takes into account an R_0 dependence with A (see ref. [15]).

From (11), the principal curvatures of the nucleus in the emission point of the alpha particle are calculated; for prolate nuclei (Fig. 1 (a))

$$k_N^x = k_N^y = \frac{1 + 5B + 4B^2}{C(1+B)^3} \tag{14}$$

and for oblate nuclei (Fig. 1 (b))

$$k_N^x = \frac{1 - 4B + (7/4)B^2}{C\left(1 - (1/2)B\right)^3}, \quad k_N^y = \frac{1}{C\left(1 - (1/2)B\right)} \tag{15}$$

where

$$B = \sqrt{\frac{5}{4\pi}}\beta_2 \tag{16}$$

The curvature of the alpha particle is equal to the inverse of its radius, in this case, the central radius; we take its sharp radius as $R_\alpha = 1.671$ fm. The universal proximity potential [17] was obtained from the Thomas-Fermi model with the inclusion of a momentum dependent nucleon-nucleon interaction potential; it reads:

$$\Phi(z/b) = -1.7817 + 0.9270 z/b + 0.0169 (z/b)^2 - $$
$$- 0.05148 (z/b)^3$$

for

$$0 \le z/b \le 1.9475 \tag{17. a}$$

and

$$\Phi(z/b) = -4.41 e^{-\frac{z}{0.7176b}}$$

for

$$1.9475 \le z/b \tag{17. b}$$

Coulomb Energy

The Coulomb interaction for two axially symmetric nuclei with quadrupole deformations can be expressed analytically [18]; in the case of the nucleus and the alpha particle we have

$$V_c(r;\theta) = 2Ze^2 \Bigg[F^{(0)}(r) + F_2^{(1)}(r) Y_2^0(\theta)\beta_2 + \tag{18}$$
$$\left(\frac{\sqrt{5}}{7}\frac{1}{\sqrt{\pi}} F_2^{(2)}(r) Y_2^0(\theta) + \frac{3}{7}\frac{1}{\sqrt{\pi}} F_4^{(2)}(r) Y_4^0(\theta)\right)\beta_2^2 \Bigg]$$

where the $F\lambda^{(n)}(r)$ are form factors and θ is the angle between the nucleus symmetry axis and the direction of relative motion (see Fig. 1). The form factors are:

$$F^{(0)}(r) = \frac{1}{r}, \quad F_2^{(1)}(r) = \frac{3}{5}\frac{C^2}{r^3}, \tag{19}$$
$$F_2^{(2)}(r) = \frac{6}{5}\frac{C^2}{r^3}, \quad F_4^{(2)}(r) = \frac{C^4}{r^5}$$

Centrifugal Potential

The orbital quantum number l of the emitted alpha particle is the fundamental factor in determining the centrifugal barrier

$$V_l(r)=\frac{\hbar^2 l(l+1)}{2\mu r^2} \tag{20}$$

In this paper is considered that both the parent nucleus and the residual nucleus are in the ground state, therefore, for even-even nuclei l = 0. For odd and odd-odd nuclei, from the classical definition of angular momentum, we can make an argument that leads to estimate a maximum value for l (l_{max}), whose fundamental idea is that the impact parameter of the alpha particle can not be greater than the radius of the emitter nucleus; from here we obtain

$$\frac{\hbar^2 l(l+1)}{2\mu}\leq Q_\alpha R_N^2 \tag{21}$$

Masses and Deformations in the Ground State

The masses of nuclei in the ground state were calculated by the macro-microscopic method, i.e. the shell correction method of Strutinsky [19]. The macroscopic part of the calculation was performed by means of a version of the liquid drop model which takes into account the finite range of the nuclear forces (FRLDM) [20, 21]; the two center shell model (TCSM) [22] was used for the microscopic part of the calculation. The energy of the system depends on five parameters (see Fig. 2); fixing three of them (ε = 1, η = 0 and $\delta_1 = \delta_2 = \delta$) a three-dimensional potential surface whose minimum point corresponds to the ground state can be constructed [23]. The deformation parameter β_2 was obtained from (11) once the minimum energy state was found and the shape of the nucleus was known.

$$\beta_2=\sqrt{4\pi}\,\frac{\int_0^\pi R_N(\theta)Y_2^0(\theta)sen\theta en}{\int_0^\pi R_N(\theta)Y_0^0(\theta)sen\theta en} \tag{22}$$

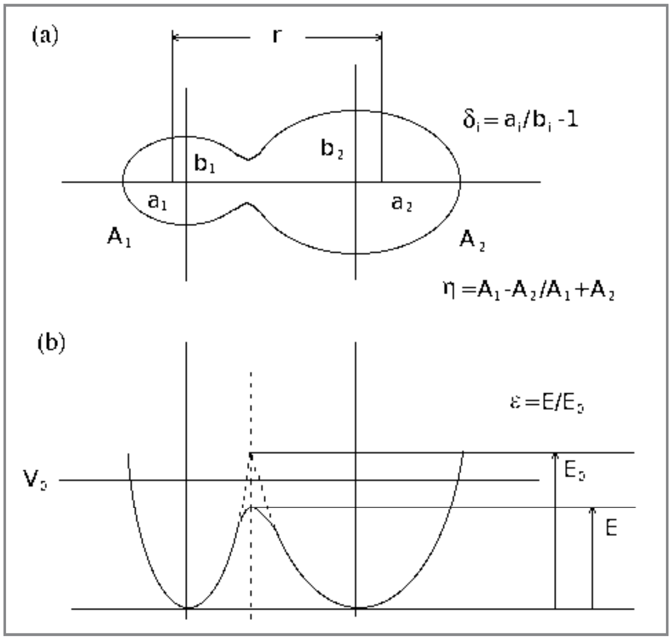

Figure 2. Parameters of the Two Center Shell Model. (a) Shape of the system of two interacting nuclei. (b) Potential energy of the system. The five parameters of the model are the elongation *r*, the mass asymmetry η, the fragments deformations δ_1 and δ_2, and the neck parameter ε.

Results and Discussion

Even-even nuclei

Figure 3 compares calculated half-lives with experimental values obtained at JIRN, of the chains of nuclei ^{294}118 and ^{292}116 [3]. It shows the results obtained from the Viola-Seaborg formula (VSS) [4] using masses and deformations calculated by means of the TCSM; it shows too the results obtained from the formalism of the barrier penetration theory (BPT), using masses and deformations calculated by means of the TCSM and also using the masses and deformations reported by Möller [21]. Parameters of the VSS formula were taken from [7]. The best result is reached for the nuclei ^{288}114 and ^{286}114, in which both, BPT and VSS calculations using TCSM masses and deformations, are in very good agreement with experiment. In the case of ^{290}116, the result from Möller is better, as in the case of ^{292}116, but for the last nucleus, BPT calculation differs from the result of VSS formula for TCSM, what indicates that there is something wrong with the TCSM deformation. In the method to find the masses and deformations of nuclei, there is a probability for a local minimum to be found in the search for a global minimum, and it could have similar energy but different deformation, and so distorts the results.

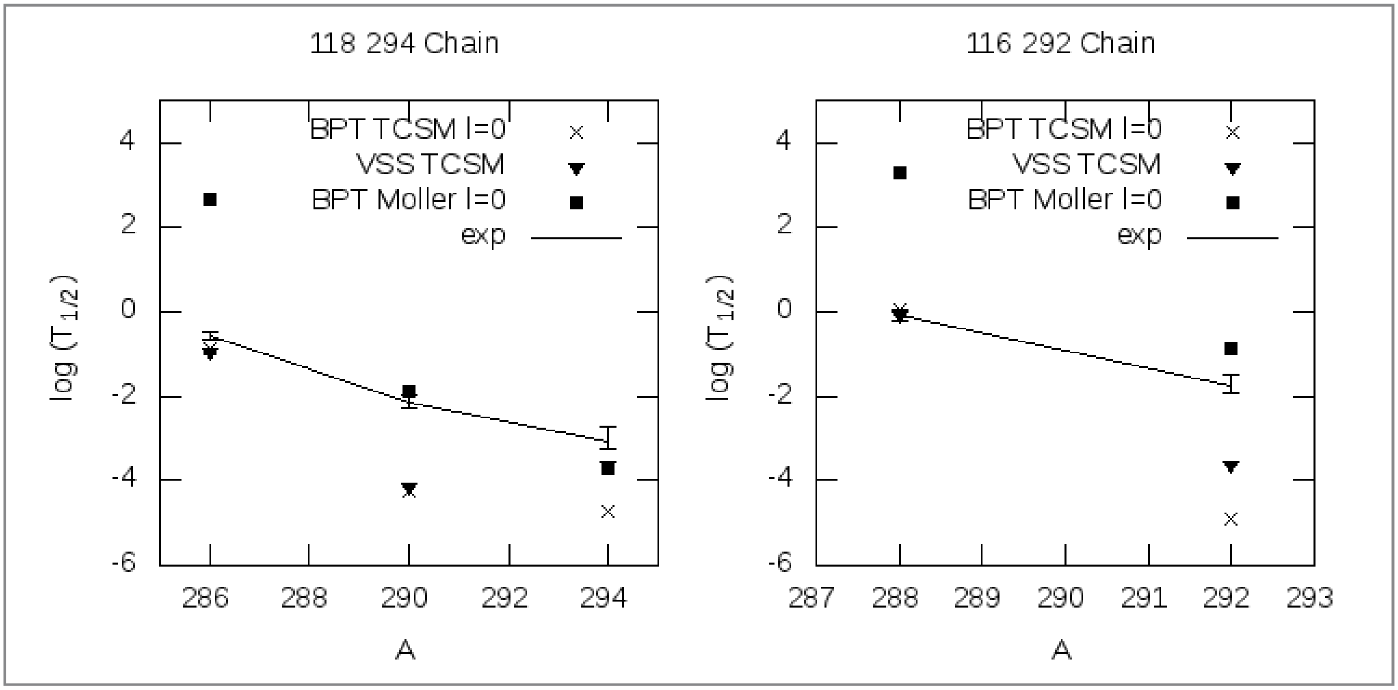

Figure 3. Comparison of VSS and BPT calculations with experiment, for $^{294}118$ and $^{292}116$ chains. TCSM and Möller masses and deformations are used.

Odd and Odd-odd Nuclei

Figure 4 compares calculated half-lives with experimental values [3] of the chains of nuclei $^{294}117$, $^{293}117$, $^{293}116$, $^{291}116$, $^{288}115$, $^{287}115$ and $^{282}113$. BPT calculations using TCSM masses and deformations were performed with l = 0 and with $l = l_{max}$; the results from the VSS formula and from BPT calculations using Möller masses and deformations with l = 0 also appear in this figure. In general, BPT calculation for l = 0 differs from VSS formula more widely than in the case of even-even nuclei, because in general, l has nonzero value. As can be seen, in general, the value given by VSS is included in the range determined by the variation of l. The previous results are in good enough agreement with experiment, with the exception of a few, for example, some isotopes of meitnerium, taking into account the margin of error that causes the variation of angular momentum.

Taking into account all of the nuclei of the chains mentioned so far (including even-even nuclei), the standard deviation s of the calculated half-lives with respect to the experimental ones can be taken as a way of comparison

$$\sigma = \left[\frac{1}{N-1} \sum_{i=1}^{N} \left(\log_{10} T_{1/2\mathrm{i}} - \log_{10} T_{1/2\mathrm{iexp}} \right)^2 \right]^{(1/2)} \quad (22)$$

In Möller case σ = 2.51 (σ = 0.82 for even-even nuclei) and in TCSM case σ = 1.61 (σ = 0.70 for even-even nuclei). In all cases we take values for *l = 0*.

Conclusions

A method for obtaining alpha-decay half-lives which is based in the WKB approximation was developed in the present work. This method takes into account the cuadrupole deformation parameter, which has a significant roll in the half-life value, as was seen in section 3.1. If good enough theoretical values of masses and deformations are used for calculations, the present method can be used as an additional way to predict or confirm experimental results in the region of superheavy nuclei.

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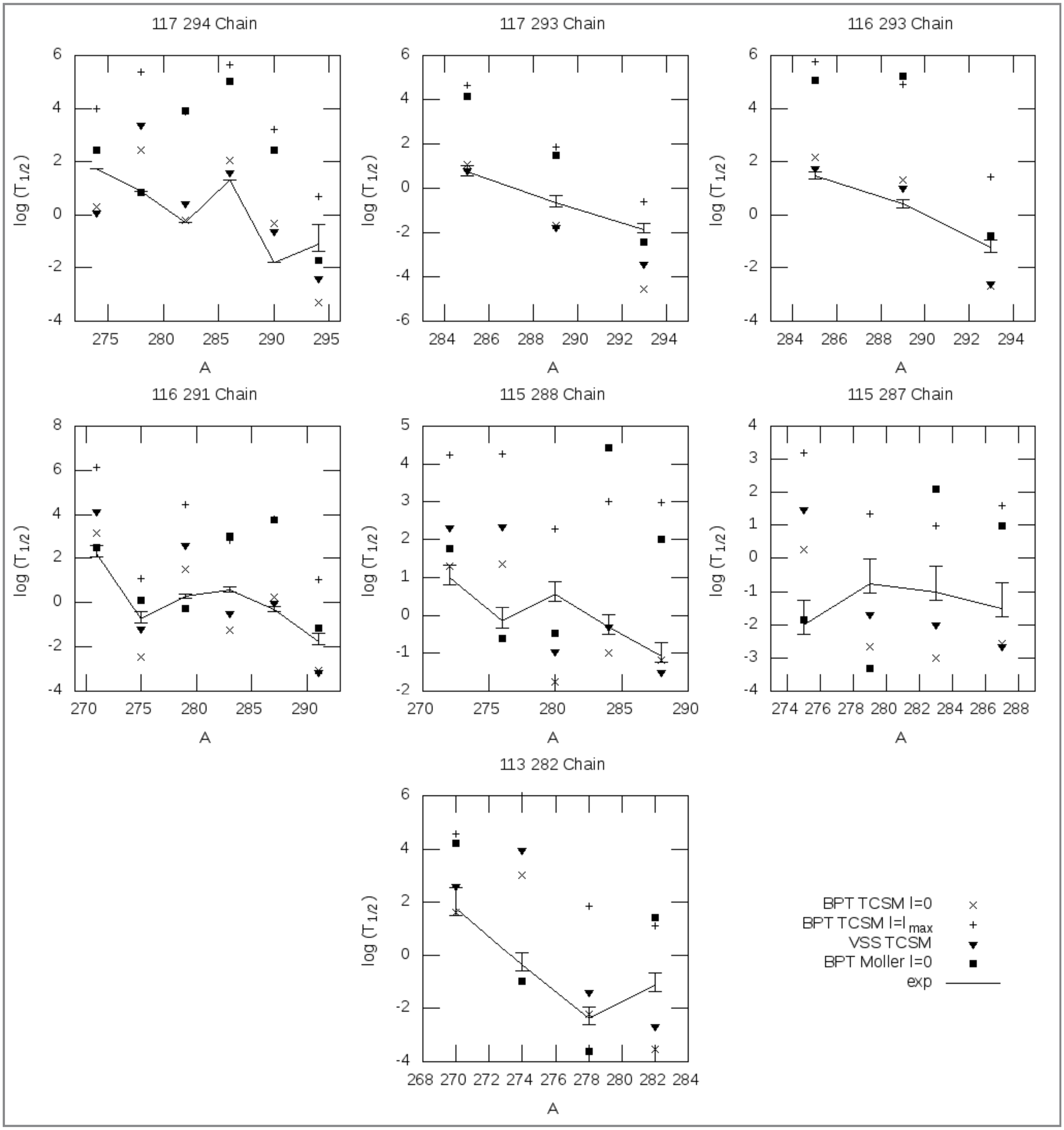

Figure 4. Comparison of VSS and BPT calculations with experiment, for $^{294}117$, $^{293}117$, $^{293}116$, $^{291}116$, $^{288}115$, $^{287}115$, and $^{282}113$ chains. TCSM and Möller masses and deformations are used.

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